Artificial Intelligence

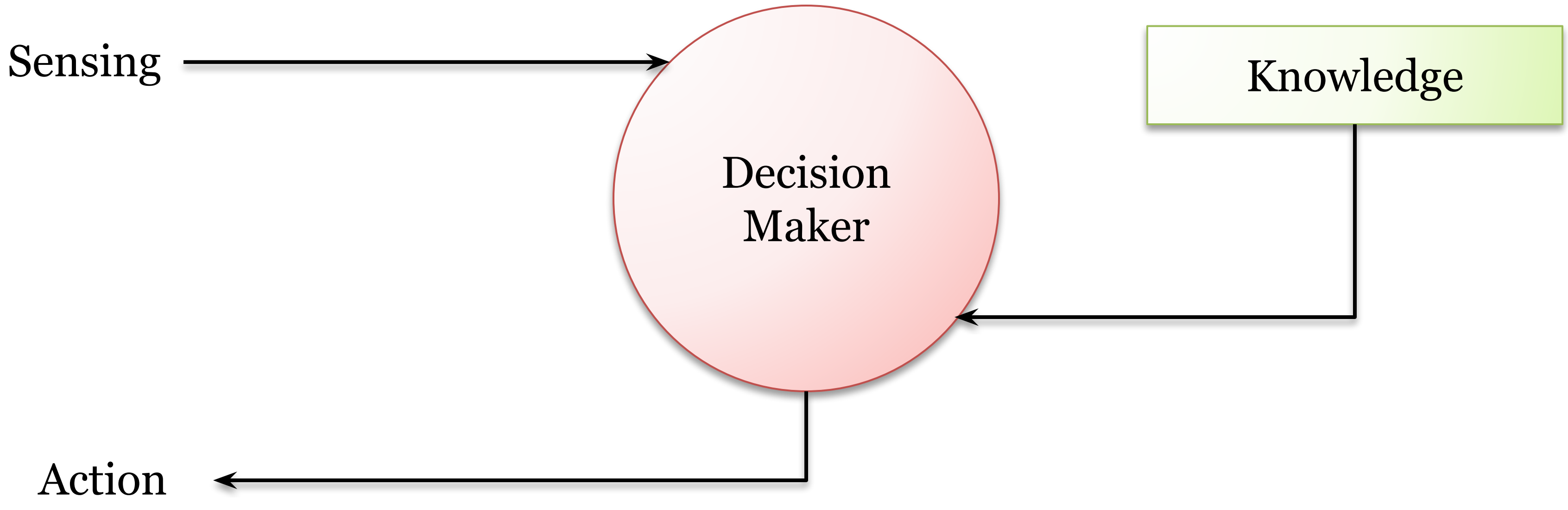
Lecture
Knowledge Representation
Propositional Logic

Knowledge Representation and Reasoning

Intelligent agents should have capacity for:

- **Perceiving**, that is, acquiring information from environment.
- **Knowledge Representation**, that is, representing its understanding of the world.
- **Reasoning**, that is, inferring the implications of what it knows and of the choices it has, and
- **Acting**, that is, choosing what it want to do and carry it out.

Knowledge and Intelligent



Knowledge Representation and Reasoning

Logic → Propositional Logic

Logic → Predicate Logic

Rule Based → e.g. if …then

SemanticNet → Knowledge Graph

Frames → Slots and fillers

Scripts

```
                                     entails
Representation:     Sentences --------------> Sentences
                      |                          |
                      |                          |
                      | Semantics                | Semantics
                      | refer to                 | refer to
                      |                          |
                      \/       follows           \/
World:              Facts ------------------> Facts
```

Logic is a Formal Language

Propositions

- Rajiv is intelligent.
- Rajiv is hardworking.
- If Rajiv is intelligent and Rajiv is hardworking then Rajiv scores high marks

Proposition: Anything that we use in our day to day statement

Elements of propositional logic

- Rajiv is intelligent.
- Rajiv is hardworking.

- **Objects** and **Relations or Functions**

Rajiv

intelligent
hardworking

- Rajiv is intelligent.
- Rajiv is hardworking.

↓

Propositions

Also Intelligent-Rajiv can be proposition.

A **proposition** (Statement) can be **true** or **false**.

- **Logic**
  - is the study of the logic relationships between objects and
  - forms the basis of all mathematical reasoning and all automated reasoning

Towards the syntax

- Let P stand for "Rajiv is intelligent"
- Let Q stand for "Rajiv is hardworking"

- What does $P \wedge Q$ (P **and** Q) mean?
- What does $P \vee Q$ (P **or** Q) mean?
- $P \wedge Q$, $P \vee Q$ are compound proposition

Propositional Logic

- In **propositional logic (PL)** an user defines a set of propositional symbols, like *P* and *Q*. User defines the semantics of each of these symbols. For example,
  - P means "It is hot"
  - Q means "It is humid"
  - R means "It is raining"
- **Set of logical operators**

$\wedge$	and	[conjunction]
$\vee$	or	[disjunction]
$\rightarrow$	implies	[implication / conditional]
$\leftrightarrow$	is equivalent	[bi-conditional]
$\neg$	not	[negation]

- **Logical Constant**: **TRUE (T)** and **FALSE (F)**

What is Propositional Logic?

1. Sometimes it is called “Sentential Logic” or “Statement Logic”
2. Deals with logical relationship between propositions (statements, Sentences, assertions, …) taken as whole

P = Rajiv is intelligent

3. Propositional logic is interested in how the truth value of “**compound claims**” depends on the truth value of the individual claims.

True / False

"Rajiv is intelligent and Rajiv is hardworking."

True / False
"Rajiv is intelligent"

True / False
"Rajiv is Hardworking"

4. Basic compound claims

P and Q
P or Q
If P then Q
not-P

Well-formed formula (wff)

- A **sentence** (also called a formula or well-formed formula or wff) is defined as:
  - Each symbol (a proposition or a constant) is a sentence
  - If S is a sentence and T is a sentence, then
    - (S) is a sentence
    - $\neg S$ is a sentence
    - $(S \lor T)$ is a sentence
    - $(S \land T)$ is a sentence
    - $(S \rightarrow T)$ is a sentence

Example wffs

- P
- True
- $P \wedge Q$
- $(P \vee Q) \rightarrow R$
- $(P \wedge Q) \vee R \rightarrow S$
- $\neg(P \vee Q)$
- $\neg(P \vee Q) \rightarrow R \wedge S$

Logical Connective: Logical And ($\wedge$)

Connective: Conjunction (symbol $\wedge$)

The logical connective And is true only when both of the propositions are true. It is also called a conjunction

Truth table

P	Q	P$\wedge$Q
T	T	T
T	F	F
F	T	F
F	F	F

<u>Logical Connective: Logical Or</u> (⋁)

<u>Connective</u>: Disjunction (symbol V)

The logical <u>disjunction</u>, or logical Or, is true if one or both of the propositions are true.

Truth table

P	Q	P⋁Q
T	T	T
T	F	T
F	T	T
F	F	F

Logical Connective: Negation (¬)

¬*p*, the negation of a proposition *p*, is also a proposition

Truth table

P	¬P
T	F
F	T

Implication →

Connective: Implication (symbol →)

- **Definition:** Let p and q be two propositions. The implication $p \rightarrow q$ is the proposition that is false when p is true and q is false and true otherwise
  - p is called the hypothesis, antecedent, premise
  - q is called the conclusion, consequence

p is a sufficient condition for q (p is sufficient for q)
q is a necessary condition for p (q is necessary for p)

Example: Implication →

- "If A then B" is false when A is true and B is false, and it is true otherwise.
- Note: **A → B is true if A is false, regardless of the truth of B**
- Example: If Ms. X passes the exam, then she will get the job
- Here B is *She will get the job* and A is *Ms. X passes the exam.*
- The statement states that Ms. X will get the job **if** a certain condition (passing the exam) is met; it says nothing about what will happen if the condition is not met. If the condition is not met, the truth of the conclusion cannot be determined; the conditional statement is therefore considered to be vacuously true, or true by default.

Truth tables

If . . . then

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

P □ q can be written as $(\neg p \vee q)$

Example

- P means "It is hot"
- Q means "It is humid"
- R means "It is raining"

Examples of PL sentences:

- $(P \land Q) \rightarrow R$ (here meaning "If it is hot and humid, then it is raining")
- $Q \rightarrow P$ (here meaning "If it is humid, then it is hot")

Equivalence ↔

<u>Connective</u>: Equivalence (symbol ↔)

$p \leftrightarrow q$ is the proposition that is true when p and q have the same truth values. It is false otherwise.

- P ↔ Q
- It is true if both A and B have the same truth values.
- It is false if A and B have opposite truth values.

Example:

- If two sides of a triangle are equal then two base angles of the triangle are equal.
- Can be represented as two sentences:
  - **(P → Q) ∧ (Q → P)**

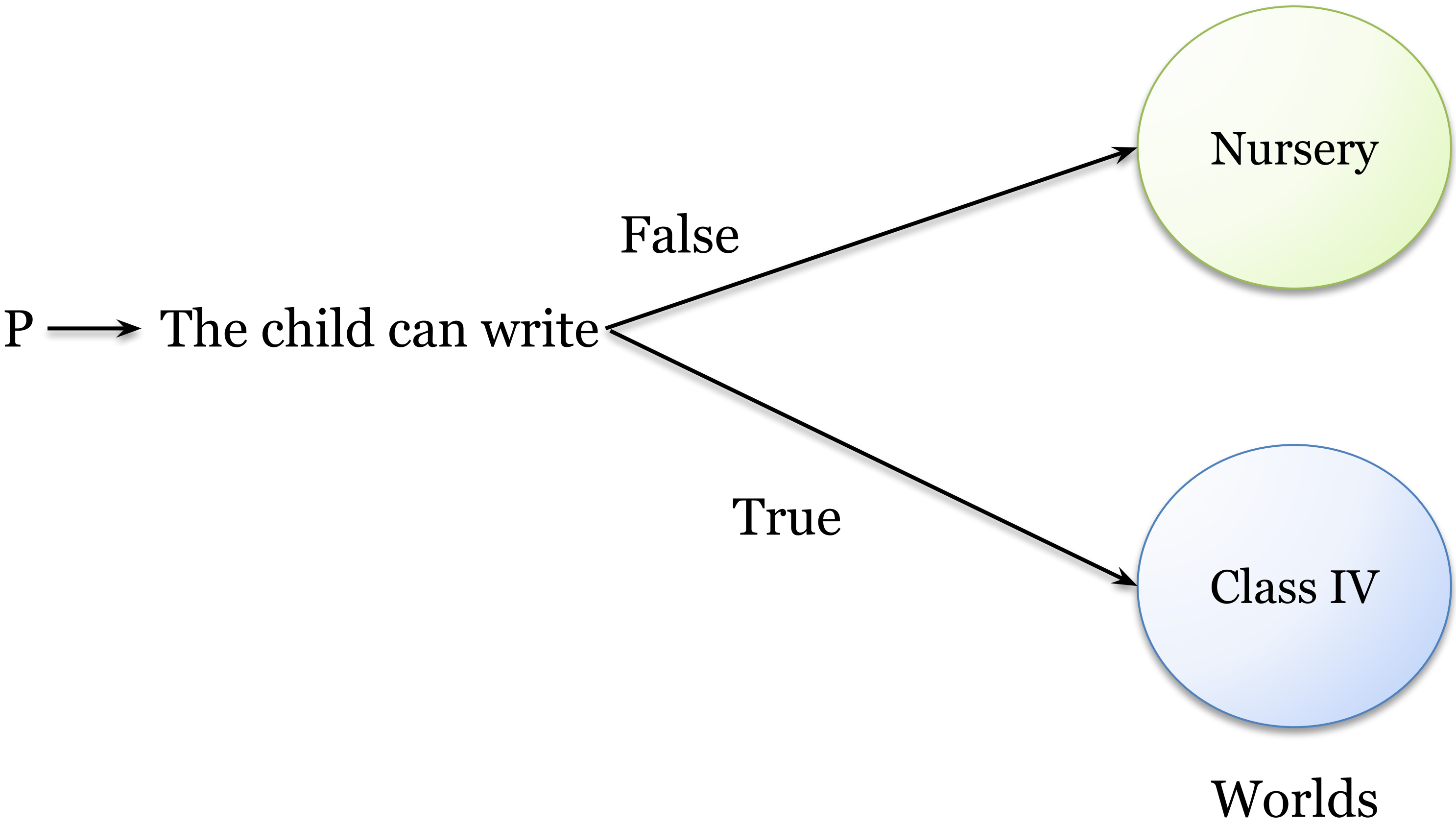
$P \leftrightarrow Q$

P	Q	P→Q	Q→P	(P→Q) ∧ (Q→P) P↔Q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

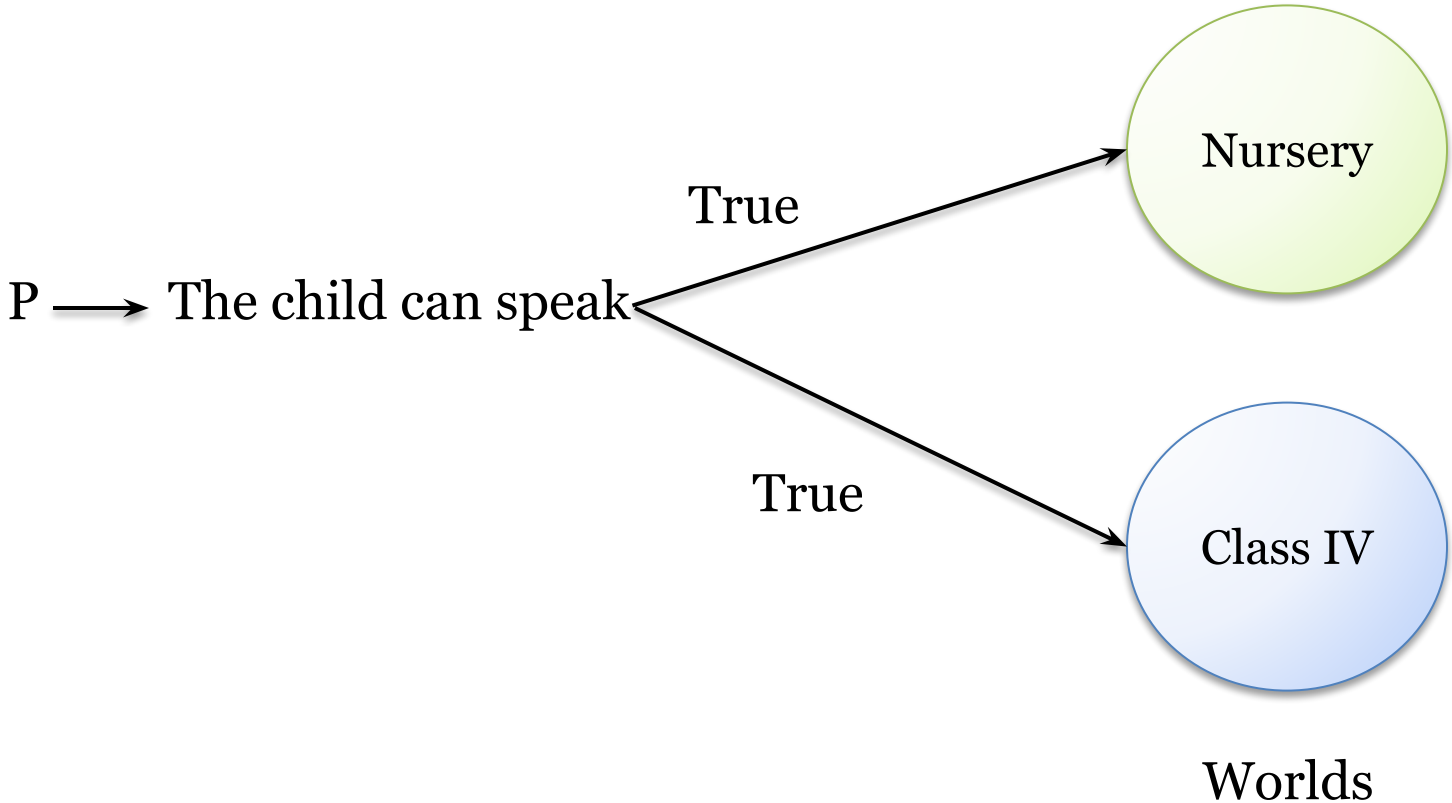
What does a wff mean

- Interpretation in a world
- When we interpret a sentence in a **world** we assign meaning to it and it evaluates to either “TRUE” or “FALSE”

Example



Example



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So how do we get the meaning?

- Sentence can be compound proposition
- Interpret each atomic proposition in the **same worlds**
- Assign truth values to each interpretation
- Compute the truth value of the compound proposition

Example

- P: Rajiv likes Avijit
- Q: Ram knows Suman

- **World**: Rajiv and Avijit are friends and Ram and Suman are known to each other.

- P = T, Q=T
- **$P \wedge Q$ = ?**
- **$P \wedge (\neg Q)$= ?**

Question

- If P is true and Q is true, then are the following true or false?
  - $P \square Q$
  - $(\neg P \vee Q) \square Q$
  - $(\neg P \vee Q) \square P$

Tautology and Contradiction

- Letters like P, Q, R, S etc. are used for representing wffs
  - $[(A \vee B) \wedge C'] \rightarrow A' \vee C$ can be represented by $P \rightarrow Q$ where
  - P is the wff $[(A \vee B) \wedge C']$ and Q represents $A' \vee C$
- Definition of tautology:
- A wff that is intrinsically true, i.e. no matter what the truth value of the statements that comprise the wff.
  - e.g. It will rain today or it will not rain today ($A \vee A'$)
  - $P \leftrightarrow Q$ where P is $A \rightarrow B$ and Q is $A' \vee B$
- Definition of a contradiction:
- A wff that is intrinsically false, i.e. no matter what the truth value of the statements that comprise the wff.
  - e.g. It will rain today and it will not rain today ($A \wedge A'$)
  - $(A \wedge B) \wedge A'$
- Usually, tautology is represented by 1 and contradiction by 0

Questions

- Express the following English Statements in the language of propositional logic:
  - It rains in July
  - If it rains today and Ram does not carry umbrella he will be drenched.

Thank You!

Any Questions?